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Cambridge International A Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **May/June 2021**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

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This document consists of **19** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

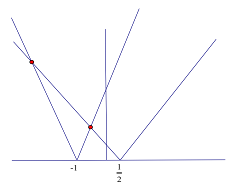
DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

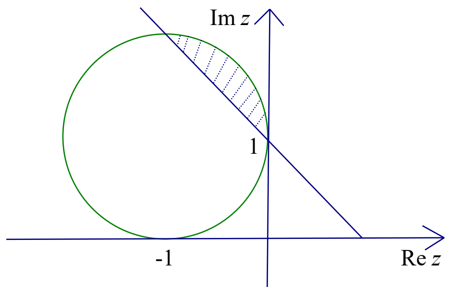
FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	State or imply non-modular inequality $(2x-1)^2 < 3^2(x+1)^2$, or corresponding quadratic equation	**B1**	e.g. $5x^2+22x+8=0$ Allow recovery from 'invisible brackets' on RHS
	Form and solve a 3-term quadratic in x	**M1**	
	Obtain critical values $x = -4$ and $x = -\frac{2}{5}$	**A1**	
	State final answer $x < -4,\ x > -\frac{2}{5}$	**A1**	Do not condone $\leqslant$ for $<$, or $\geqslant$ for $>$ in the final answer. Allow 'or' but not 'and'. $-\frac{2}{5} < x < -4$ scores A0. Accept equivalent forms using brackets e.g. $x \in (-\infty, -4) \cup (-0.4, \infty)$
	Alternative method for Question 1		
	Obtain critical value $x = -4$ from a graphical method, or by solving a linear equation or linear inequality	**B1**	
	Obtain critical value $x = -\frac{2}{5}$ similarly	**B2**	
	State final answer $x < -4,\ x > -\frac{2}{5}$	**B1**	Do not condone $\leqslant$ for $<$, or $\geqslant$ for $>$ in the final answer. Allow 'or' but not 'and'. $-\frac{2}{5} < x < -4$ scores A0. Accept equivalent forms using brackets e.g. $x \in (-\infty, -4) \cup (-0.4, \infty)$
		4	

Question	Answer	Marks	Guidance
2	Show a circle with centre $-1+i$.	**B1**	Need some indication of scale or a correct label. Could just be mark(s) on the axes
	Show a circle with radius 1 and centre not at the origin (or relevant part thereof).	**B1**	
	Show correct half line from 1(or relevant part thereof) .	**B1**	
	Shade the correct region on a correct diagram.	**B1**	
		**4**	N.B. If they have very different scales on *their* 2 axes the diagram must match *their* scale - the 'circle' should be an ellipse. Allow freehand diagrams with clear correct intention.

Question	Answer	Marks	Guidance
3(a)	State or imply $\ln x = \ln A - y\ln 3$	**B1**	$\left(y = -\dfrac{1}{\ln 3}\ln x + \dfrac{\ln A}{\ln 3}\right)$
	State that the graph of y against $\ln x$ has an equation that is *linear* in y and $\ln x$, or has an equation of the standard form '$y = mx + c$' and is thus a straight line	**B1**	Must be a correct statement. Accept if the 2 equations are written side by side with no comment. An equation with $y\ln 3$ should be compared with the form $py + q\ ln\ x = c$.
	State that the gradient is $-\dfrac{1}{\ln 3}$	**B1**	OE. Exact answer required. ISW after a correct statement.
		3	

Question	Answer	Marks	Guidance
3(b)	Substitute ln x = 0, y = 1.3 and use correct method to solve for A	M1	$(\ln A = 1.3\ln 3)$ Follow *their* equation in y and $\ln x$. Must be substituting ln$x$ = 0, not x = 0. ln 0 'used' in the solution scores M0A0.
	Obtain answer A = 4.17 only	A1	Must be 2 d.p. as specified in question
		2	

Question	Answer	Marks	Guidance
4	Commence integration and reach $ax\tan^{-1}\frac{1}{2}x + b\int x\frac{1}{c+x^2}\mathrm{d}x$	*M1	OE. Denominator might be $1+\frac{x^2}{4}$ or $2+\frac{x^2}{2}$.
	Obtain $x\tan^{-1}\left(\frac{1}{2}x\right) - \int x.\ \frac{2}{4+x^2}\ \mathrm{d}x$	A1	OE
	Complete integration and obtain $x\tan^{-1}\left(\frac{1}{2}x\right) - \ln\left(4+x^2\right)$	A1	OE e.g. with $\ln\left(1+\frac{x^2}{4}\right)$
	Substitute limits correctly in an expression of the form $px\tan^{-1}x + q\ln\left(c+x^2\right)$	DM1	$2\tan^{-1}1 - \ln 8 + \ln 4$ OE
	Obtain final answer $\frac{1}{2}\pi - \ln 2$	A1	OE exact answer. Needs a value for $\tan^{-1}1$ and a single log term
	Alternative method for Question 4		
	Use the substitution $\theta = \tan^{-1}\frac{x}{2}$ to obtain $\lambda\int 2\theta\sec^2\theta\,\mathrm{d}\theta$ and reach $p\theta\tan\theta + q\int\tan\theta\,\mathrm{d}\theta$	*M1	

Question	Answer	Marks	Guidance
4	Obtain $2\theta\tan\theta - 2\int\tan\theta\,d\theta$	A1	OE
	Complete integration and obtain $2\theta\tan\theta + 2\ln(\cos\theta)$	A1	OE
	Substitute correct limits correctly in an expression of the form $r\theta\tan\theta + s\ln(\cos\theta)$	DM1	Limits should be $\frac{\pi}{4}$ and 0. Limits must be in radians.
	Obtain final answer $\frac{1}{2}\pi - \ln 2$	A1	OE exact answer. Need values for trig. functions and a single log term.
		5	

Question	Answer	Marks	Guidance
5	Square $a + \mathrm{i}b$, use $\mathrm{i}^2 = -1$ and equate real and imaginary parts to 10 and $-4\sqrt{6}$ respectively	**M1**	
	Obtain $a^2 - b^2 = 10$ and $2ab = -4\sqrt{6}$	**A1**	Allow $2ab\mathrm{i} = -4\sqrt{6}\mathrm{i}$
	Eliminate one unknown and find an equation in the other	**M1**	Must be sensible algebra e.g. use of $\sqrt{a^2 - b^2} = a - b$ socres M0
	Obtain $a^4 - 10a^2 - 24[=0]$, or $b^4 + 10b^2 - 24[=0]$, or 3-term equivalent	**A1**	Or equivalent horizontal equation from correct work
	Obtain final answers $\pm\left(2\sqrt{3} - \sqrt{2}\mathrm{i}\right)$, or exact equivalents	**A1**	e.g. $\pm\left(\sqrt{12} - \sqrt{2}\mathrm{i}\right)$ from correct work
	Alternative method for Question 5		
	Use the correct method to find the modulus and argument of $\sqrt{u}$	**M1**	
	Obtain modulus $\sqrt{14}$	**A1**	
	Obtain argument $\tan^{-1}\dfrac{-1}{\sqrt{6}}$ using an exact method	**A1**	e.g. by using half angle formula which gives $2\sqrt{6}t^2 - 10t - 2\sqrt{6} = 0$
	Convert to the required form	**M1**	$\pm\sqrt{14}\left(\dfrac{\sqrt{6}}{\sqrt{7}} - \dfrac{1}{\sqrt{7}}\mathrm{i}\right)$ This mark is available if working in decimals
	Obtain answers $\pm\left(2\sqrt{3} - \sqrt{2}\mathrm{i}\right)$, or exact equivalents	**A1**	e.g. $\pm\left(\sqrt{12} - \sqrt{2}\mathrm{i}\right)$
		5	

Question	Answer	Marks	Guidance
6(a)	Express the LHS in terms of $\cos 2\theta$ and $\sin 2\theta$	**B1**	e.g. $\frac{1}{\sin 2\theta} - \frac{\cos 2\theta}{\sin 2\theta}$
	Use correct double angle formulae to express the LHS in terms of $\cos\theta$ and $\sin\theta$	**M1**	e.g. $\frac{1-\left(1-2\sin^2\theta\right)}{2\sin\theta\cos\theta}$
	Obtain $\tan\theta$ from correct working	**A1**	AG
	Alternative method for Question 6(a)		
	Express the LHS in terms of $\sin 2\theta$ and $\tan 2\theta$	**B1**	
	Use correct double angle formulae to express the LHS in terms of $\cos\theta$ and $\sin\theta$	**M1**	e.g. $\frac{1}{2\sin\theta\cos\theta} - \frac{1-\frac{\sin^2\theta}{\cos^2\theta}}{2\frac{\sin\theta}{\cos\theta}}\left(=\frac{4\sin^2\theta}{4\sin\theta\cos\theta}\right)$
	Obtain $\tan\theta$ from correct working	**A1**	AG
	Alternative method for Question 6(a)		
	Express the LHS in terms of $\sin 2\theta$ and $\tan 2\theta$	**B1**	
	Use correct t substitution or rearrangement of $\sin 2\theta$ in terms of $\sec^2 2\theta$ and $\tan\theta$ to express the LHS in terms of $\tan\theta$.	**M1**	$\left(\frac{\sec^2\theta}{2\tan\theta} - \frac{1-\tan^2\theta}{2\tan\theta} = \right)\frac{1+\tan^2}{2\tan} - \frac{1-\tan^2}{2\tan}$
	Obtain $\tan\theta$ from correct working	**A1**	AG
		3	

Question	Answer	Marks	Guidance
6(b)	State integral of the form $\mp \ln \cos\theta$ or $\pm \ln \sec\theta$	*M1	$[-\ln \cos\theta]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$ OE
	Use correct limits correctly and insert exact values for the trigonometric ratios	DM1	Need to see evidence of the substitution
	Obtain a correct expression, e.g. $-\ln\frac{1}{2} + \ln\frac{1}{\sqrt{2}}$	A1	
	Obtain $\frac{1}{2}\ln 2$ from correct working	A1	AG (must see an intermediate step)
		4	

Question	Answer	Marks	Guidance
7	State equation $\frac{dy}{dx} = k\frac{y}{\sqrt{x+1}}$	B1	OE. Must be a differential equation.
	Separate variables correctly for *their* differential equation and integrate at least one side	*M1	$\int \frac{1}{y}\,dy = \int \frac{k}{\sqrt{x+1}}\,dx$
	Obtain $\ln y$	A1	Allow M1A1A1 if they have assumed $k = 1$ or are working with an incorrect value for k
	Obtain $2[k]\sqrt{x+1}$	A1	
	Use (0, 1) **and** (3, e) in an expression containing $\ln y$ and $\sqrt{x+1}$ and a constant of integration to determine k and/or a constant of integration c (or use (0, 1), (3, e) and (x, y) as limits on definite integrals)	DM1	If remove logs before finding the constant of integration then the constant must be of the correct form.
	Obtain $k = \frac{1}{2}$ and $c = -1$	A1	OE. $\left(\ln y = \sqrt{x+1} - 1\right)$ Their value of c will depend on where c is in their equation and whether they are working with $\frac{1}{k}\ln y$. The value of k must be consistent with what they integrated.
	Obtain $y = \exp\left(\sqrt{x+1} - 1\right)$	A1	NFWW, OE, ISW.
		7	

Question	Answer	Marks	Guidance
8	Use correct product (or quotient) rule	**M1**	At least 3 of 4 terms correct
	Obtain $\frac{dy}{dx} = -5e^{-5x}\tan^2 x + 2e^{-5x}\tan x\sec^2 x$	**A1**	OE.
	Equate *their* derivative to zero, use $\sec^2 x = 1 + \tan^2 x$ and obtain an equation in $\tan x$	**M1**	
	Obtain $2\tan^2 x - 5\tan x + 2 = 0$	**A1**	Allow $2\tan^3 x - 5\tan^2 x + 2\tan x = 0$
	State answer $x = 0$	**B1**	From correct derivative.
	Solve a 3 term quadratic in $\tan x$ and obtain a value of x	**M1**	Must be in radians
	Obtain answer, e.g. 0.464	**A1**	Must be 3 d.p. as specified in the question. Allow A1A0 if both values given to 2 d.p. or > 3 d.p.
	Obtain second non-zero answer, e.g. 1.107 and no other in the given interval	**A1**	
	Alternative method for Question 8		
	Use correct product (or quotient) rule	**M1**	At least 3 of 4 terms correct
	Obtain $\frac{dy}{dx} = -5e^{-5x}\tan^2 x + 2e^{-5x}\tan x\sec^2 x$	**A1**	OE
	Equate *their* derivative to zero and obtain an equation in $\sin x$ and $\cos x$	**M1**	
	Obtain $5\cos x\sin x = 2$	**A1**	Or simplified equivalent (i.e. cancelled)
	State answer $x = 0$	**B1**	From correct derivative.
	Use double angle formula or square both sides and solve for x	**M1**	Or equivalent method. Must be in radians.
	Obtain answer, e.g. 0.464	**A1**	Must be 3 d.p. as specified in the question. Allow A1A0 if both values given to 2 d.p. or > 3 d.p.
	Obtain second non-zero answer, e.g. 1.107 and no other in the given interval	**A1**	
		8	

Question	Answer	Marks	Guidance
9(a)	State or imply the form $\dfrac{A}{2+x}+\dfrac{B+Cx}{3+x^2}$	**B1**	
	Use a correct method for finding a constant	**M1**	SOI
	Obtain one of $A = 4$, $B = 1$ and $C = -2$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	ISW
		5	
9(b)	Use correct method to find the first two terms of the expansion of $(2+x)^{-1}$, $\left(1+\frac{1}{2}x\right)^{-1}$, $\left(3+x^2\right)^{-1}$ or $\left(1+\frac{1}{3}x^2\right)^{-1}$	**M1**	Allow unsimplified but not if still including ${}^nC_r$
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	**A1 FT** **A1 FT**	$2\left(1-\frac{1}{2}x+\left(\frac{1}{2}x\right)^2....\right)$ $+\frac{1}{3}(1-2x)\left(1-\frac{1}{3}x^2....\right)$ The FT is on *their* A, B and C
	Multiply out, up to the terms in x^2, by $B + Cx$, where $BC \neq 0$	**M1**	Allow with B and C as implied in part **(b)**
	Obtain final answer $\frac{7}{3}-\frac{5}{3}x+\frac{7}{18}x^2$	**A1**	Or equivalent in form $p+qx+rx^2$. A0 if they multiply through by 18.
		5	

Question	Answer	Marks	Guidance
10(a)	State or imply $CD = 2r - 2r\cos x$	B1	
	Using correct formulae for area of sector and trapezium, or equivalent, form an equation in r and x	M1	e.g. $2\times\frac{1}{2}r^2x = \frac{0.9}{2}(2r + 2r - 2r\cos x)r\sin x$
	Obtain $x = 0.9(2 - \cos x)\sin x$	A1	AG, NFWW
		3	
10(b)	Calculate the values of a relevant expression or pair of expressions at $x = 0.5$ and $x = 0.7$	M1	Calculated for both values and correct for one value is sufficient for M1. Must be working in radians.
	Complete the argument correctly with correct values	A1	Must have sufficient accuracy to support the answer e.g. $\begin{matrix}0.5 > 0.484 \\ 0.7 < 0.716\end{matrix}$ or $\begin{matrix}0.016 > 0 \\ -0.016 < 0\end{matrix}$ or $\begin{matrix}0.96\ldots < 1 \\ 1.02\ldots > 1\end{matrix}$
		2	
10(c)	State a suitable equation, e.g. $\cos x = \left(2 - \frac{x}{0.9\sin x}\right)$	B1	If working in reverse, the first B1 is for $\frac{x}{0.9\sin x} = 2 - \cos x$
	Rearrange this as $x = 0.9\sin x(2 - \cos x)$	B1	Need to see the complete sequence of changes.
		2	

Question	Answer	Marks	Guidance
10(d)	Use the iterative process correctly at least once	**M1**	Must be working in radians
	Obtain answer 0.62	**A1**	
	Show sufficient iterations to at least 4 d.p.to justify 0.62 to 2 d.p., or show there is a sign change in the interval (0.615, 0.625)	**A1**	Allow recovery. N.B. A candidate who starts with 0.5 and stops at 0.61 or starts at 0.7 and stops at 0.63 can score M1A0A1 if they have worked to the required accuracy.
		3	
11(a)	Show that $OA = OB = \sqrt{5}$	**B1**	CWO
	Evaluate the scalar product of the correct position vectors	**M1**	e.g. $(0-1+0)$ Condone of using *AO* and/or *BO*
	Divide *their* scalar product by the product of the moduli of *their* vectors and evaluate the inverse cosine of the result	**M1**	Much reach an angle. The question asks for the use of scalar product, so alternative methods (e.g. cosine rule) are not accepted.
	Obtain answer 101.5°	**A1**	The question asks for an answer in degrees. Accept 102° or better. Mark radians (1.77) as a misread. Do not ISW: 78.5° as final answer scores A0.
		4	

Question	Answer	Marks	Guidance
11(b)	State or imply M has position vector $\mathbf{i} - \mathbf{k}$	**B1**	OE
	Taking a general point of OM to have position vector $\lambda\mathbf{i} - \lambda\mathbf{k}$, express $AP = \sqrt{7}\ OA$ as an equation in λ	***M1**	$\lambda\left(\textit{their}\ \overline{OM}\right)$
	State a correct equation in any form	**A1**	e.g. $\sqrt{(-2+\lambda)^2 + 1 + (-\lambda)^2} = \sqrt{7}\sqrt{5}$
	Reduce to $\lambda^2 - 2\lambda - 15 = 0$	**A1**	OE
	Solve a quadratic and state a position vector	**DM1**	
	Obtain answers $5\mathbf{i} - 5\mathbf{k}$ and $-3\mathbf{i} + 3\mathbf{k}$	**A1**	Accept coordinates
	Alternative method for Question 11(b)		
	State or imply that $OP = \gamma\sqrt{2}$	**B1**	
	State or imply that $\cos\frac{1}{2}AOB = \sqrt{\frac{2}{5}}$ and use cosine rule to form an equation in γ	***M1**	Allow $\cos\frac{1}{2}AOB = 0.632...$
	State a correct equation in any form	**A1**	e.g. $35 = 5 + 2\gamma^2 - 2\sqrt{5}.\gamma\sqrt{2}.\frac{\sqrt{2}}{\sqrt{5}}$
	Reduce to $\gamma^2 - 2\gamma - 15 = 0$	**A1**	OE
	Solve a quadratic and state a position vector	**DM1**	
	Obtain answers $5\mathbf{i} - 5\mathbf{k}$ and $-3\mathbf{i} + 3\mathbf{k}$	**A1**	Accept coordinates

Question	Answer	Marks	Guidance
11(b)	**Alternative method for Question 11(b)**		
	State or imply M has position vector $\mathbf{i} - \mathbf{k}$	**B1**	OE
	State or imply that $AM = \sqrt{3}$	**B1**	
	Use Pythagoras to find MP	***M1**	$MP = \sqrt{35 - (AM)^2}$
	Obtain $MP = 4\sqrt{2}$	**A1**	
	Correct method to find a position vector	**DM1**	$(\mathbf{i} - \mathbf{k}) \pm 4(\mathbf{i} - \mathbf{k})$
	Obtain answers $5\mathbf{i} - 5\mathbf{k}$ and $-3\mathbf{i} + 3\mathbf{k}$	**A1**	Accept coordinates
		6	